INTERNATIONAL MATHEMATICS
# TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 41, Northern Spring 2020 (A Level)



**Note:** Each contestant is credited with the largest sum of points obtained for three problems.

1. Does there exist a positive integer that is divisible by 2020, which contains each of the digits $0, 1, 2, \ldots, 9$ the same number of times? (4 points)

2. Three legendary knights are fighting against a dragon with multiple heads. Whenever the first knight attacks, he cuts off half of the current number of the dragon's heads plus one more head. Whenever the second knight attacks, he cuts off one third of the current number of the dragon's heads plus two more heads. Whenever the third knight attacks, he cuts off one quarter of the current number of the dragon's heads plus three more heads. They repeatedly attack the dragon in an arbitrary order so that at each step an integer number of heads is being cut off. If none of the knights is able to attack since the number of heads would become a non-integer, the dragon wins eating all of them. Will the knights be able to cut off all the dragon's heads if it has 41! heads?
**Note:** $41! = 1 \times 2 \times 3 \times \cdots \times 41$. (5 points)

3. Does there exist an $N$-gon inscribed in a circle such that all its sides have different lengths and all its angles have integer values in degrees, where

   (a) $N = 19$? (4 points)

   (b) $N = 20$? (3 points)

4. For which integers $N$ is it possible to fill the $1 \times 1$ cells of an $N \times N$ grid with real numbers so that all the integers from 1 to $2(N-1)N$ are among the sums of the numbers in each adjacent pair of cells and each integer occurs exactly once?
**Note.** Two cells are *adjacent* if they share a side. (8 points)

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**5.** Let $ABCD$ be an inscribed trapezium, such that $AB$ is parallel to $CD$ and $AB = 3CD$. Tangents to the circumcircle of $ABCD$ at points $A$ and $C$ intersect at the point $K$. Prove that $\angle KDA = 90°$. (9 points)

**6.** Petya has a deck of 36 cards, comprising 4 suits of 9 cards each. He chooses any 18 cards and gives the rest to Vasya. They play in turn. Petya puts down one of his cards first and then Vasya puts down one of his cards, both cards being face up. If Vasya can put down his card such that the two cards are of the same suit or of the same value, he gains a point. What is the maximum number of points Vasya can gain for sure no matter how Petya plays? (9 points)

**7.** Gleb starts by choosing positive integers $N$ and $a$, where $a < N$, and writes $a$ on a blackboard. Then, at each step, he divides $N$ by the last number written on the blackboard (which, at the first step is $a$), and writes the remainder on the board, which is now the last number written on the board. Gleb continues the process until finally he writes 0 on the board, where the process terminates. Is it possible for Gleb to choose $N$ and $a$, such that the sum of the numbers written on the board would be greater than $100N$? (12 points)